Fe or Co occupies the inversion centers with calcium in mirror plane positions, isostructural with monticellite $\left(\mathrm{CaMgSiO}_{4}\right)$. Attempts to synthesize $\mathrm{CaNiSiO}_{4}$ proved unsuccessful. In $\mathrm{CaMnSiO}_{4}$, the manganese moments align in antiferromagnetic chains along b with spin directions collinear with $\mathbf{c}$. The cobalt salt has a large magnetic cell similar to $\mathrm{Ni}_{2} \mathrm{SiO}_{4}$, but systematic absences suggest a ferromagnetic direct interaction. $\mathrm{CaFeSiO}_{4}$ remains paramagnetic to low temperatures.

Only superexchange interactions occur in the lithio-philite-triphylite group where the transition-metal ions are partially replaced by the smaller $\mathrm{Li}^{+}$ion. As mentioned previously, the $\mathrm{LiMPO}_{4}(\mathrm{M}=\mathrm{Mn}, \mathrm{Fe}, \mathrm{Co}, \mathrm{Ni})$ compounds possess similar magnetic structures. $\mathrm{M}-\mathrm{O}-\mathrm{M}$ superexchange interactions give rise to antiferromagnetic puckered-planes orthogonal to a. There are no direct or superexchange linkages between these planes, and it is necessary to postulate long-range interactions, such as the $\mathrm{Mn}-\mathrm{O}-\mathrm{P}-\mathrm{O}-\mathrm{Mn}$ triple exchange suggested by Mays (1963).

In $\mathrm{LiMnPO}_{4}$ and $\mathrm{Mn}_{2} \mathrm{SiO}_{4}$ the spins align along a, the direction minimizing the magnetic dipole energy. b is the preferred direction for the iron and cobalt compounds, in which spin-orbit and crystal field interactions are often decisive. The spin direction of the magnetically ordered lithiophilite-triphylite series must depend on composition since the magnetic structures of the end members differ. Susceptibility measurements on $\mathrm{LiMn}_{0.7} \mathrm{Fe}_{0.3} \mathrm{PO}_{4}$ single crystals (Bozorth \& Kramer, 1959) specify $b$ as the spin direction, like $\mathrm{LiFePO}_{4}$. The change in spin direction therefore takes place in man-ganese-rich compositions near $\mathrm{LiMnPO}_{4}$.

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# Five New Zinc Sulphide Polytypes: 10L (8 2); 14L (5 42 3); 24L (5 3) $)_{3}$; $26 L(1742$ 3) and $28 L(9559)$ 

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#### Abstract

The layer sequences of some ZnS polytypes have been determined by comparing observed X-ray reflexion intensities with those calculated by a computer. It is demonstrated that birefringence measurements facilitate these determinations, since the number of elements of the Zhdanov symbol can be known from measuring the degree of the birefringence. Differences of polytypism in SiC and ZnS are discussed briefly.


## Introduction

It has been known (Frondel \& Palache, 1950), that both natural and artificially grown ZnS crystals often exhibit polytypism. Still, very little is known about polytypism in ZnS , since even in the cases where the
existence of a polytype has been reported, usually only the periodicity of the structure is given and the actual stacking sequence is missing. The only exceptions are $6 L$ [six-layer periodicity, (3 3)], 15L (3 2) 3 (Frondel \& Palache, 1950), $8 L(44)$ and $10 L$ (5 5) (Evans \& McKnight, 1959), the symbols in parentheses denoting
the number of consecutive layers stacked in cyclic or anticyclic order (Zhdanov, 1945). The layer sequence is automatically determined by the periodicity in the other three known ZnS structures: hexagonal $2 L$ (11), cubic $3 L(3)_{\infty}$ and four-layer $\left.4 L(2) 2\right)$ (Frondel \& Palache, 1950).

One of the difficulties of structure determinations in ZnS has been the fact that in vapour-phase grown samples the regions which are crystallographically uniform have inconveniently small dimensions. In fact, X-ray work often yielded a mixture of several structures as well as one-dimensional stacking disorder (Strock \& Brophy, 1955).

The layer sequence of a polytype can be determined, in principle, by comparison of the observed X-ray diffraction intensity distribution with the intensity distributions of all geometrically possible layer sequences. However, the number of possible layer sequences increases very rapidly with the period $m$ of the polytype and therefore comparison with all possible sequences usually becomes impracticable even when a fast digital computer is used. Accordingly, for high polytypes special considerations have to be taken into account, in order to limit the number of layer sequences to be compared. In SiC , where a large number of polytypes are known, this is done by looking only for certain types of layer sequence (e.g. Krishna \& Verma, 1962). There is no reason to believe that the same types of layer sequence are of special importance in ZnS as well; it will be seen that layer sequences of other types also occur.

For the present work ZnS crystals which contain uniform regions of convenient width were available. X-ray oscillation photographs were taken of the uniform regions. The observed intensity distributions were compared with intensities calculated by an electronic computer for possible layer sequences. The number of sequences to be calculated was reduced, in the case of higher polytypes, by using a recently found correlation between birefringence and structure (Brafman \& Steinberger, 1966).

## Experimental

The crystals used were grown by sublimation. Details of growing conditions as well as a table of typical cationic impurities can be found in a previous publication (Brafman, Alexander, Fraenkel, Kalman \& Steinberger, 1964).

The crystals used were platelets, 30 to $200 \mu$ thick, the main faces being either (10.0) or (12.0) planes. Under the polarizing microscope they appeared to consist of parallel coloured strips. The strips are joined to each other along ( 00.1 ) planes. The colour changes are due to variations of the birefringence; this, in turn, is determined by the crystal structure. For the present work crystals were selected with wide regions ( $>0.1 \mathrm{~mm}$ ) of uniform birefringence; X-ray analysis showed that the regions were uniform structurally as well. The optical band gaps and the birefringence of
the crystals used have been reported previously (Brafman \& Steinberger, 1966).

X-ray oscillation photographs were taken from the uniform regions with a 60 mm diameter camera mounted on a Hilger microfocus instrument. The crystals were oscillated around the $c$ axis, which is common to all regions. The diameter of the X-ray beam emerging from the collimator was 0.1 mm . A polarizing microscope, mounted on the X-ray camera, served for adjusting the region to be investigated in the X-ray beam.

The reflexion spot intensities along (10.l) were estimated by visual comparison of the spots. This procedure gave very well reproducible estimates for nearby spots; it is, of course, somewhat less satisfactory for distant ones.

The degree of birefringence $\Delta n=n_{e}-n_{0}$, where $n_{e}$ and $n_{0}$ are the extreme values of the refractive index, was determined by using a suitable compensator coupled with thickness measurements.

## Computational methods

In case of the lower polytypes the unit translation in the $c$ direction could be determined directly from distance measurements along ( $10 . l$ ) columns. This procedure immediately yields the period $m$ of the polytype. For higher polytypes only a first estimate of $m$ can be obtained in this way, because of the experimental error of the distance measurements. In such polytypes use was made of the periodicity of (10.l) structure factors with period $m$. Corresponding spots could therefore be easily identified by looking for groups of a few spots with similar intensity distributions. Counting the number of spots between two corresponding spots yields $m$ unambiguously (Krishna \& Verma, 1963).

From a knowledge of $m$, the possible reflexion intensity distributions were calculated with the IBM 7040 computer. This was done in two steps: (a) enumeration of the possible layer sequences for the given $m$; $(b)$ calculation of the reflexion intensities.
(a) The layer sequences of an $m$-layer polytype are of the form ( $a_{1} b_{1} a_{2} b_{2} \ldots a_{k} b_{k}$ ), where the $a$ 's and $b$ 's are natural numbers denoting the number of consecutive ( 00.1 ) planes, stacked in cyclic or anticyclic order respectively (Zhdanov, 1945).
Writing $\sum_{i=1}^{k} a_{i}=a$ and $\sum_{i=1}^{k} b_{i}=b$ one has

$$
\begin{align*}
& a+b=m  \tag{1}\\
& a-b=0(\bmod 3) \tag{2}
\end{align*}
$$

Condition (2) ensures that the first and the last layers of the sequence shall not be in the same positions.

Conditions (1) and (2) yield all possible layer sequences, with many repetitions. A large part of the repetitions was avoided by imposing two further restrictions, namely

$$
\begin{align*}
& a \geq b  \tag{3}\\
& a_{1} \geq a_{j} \quad j=2, \quad \ldots k \tag{4}
\end{align*}
$$

The number of possible polytypes increases very steeply with $m$ and full enumeration becomes impracticable even for the computer. Therefore for the higher polytypes ( $26 L$ and $28 L$ ) use was made of the proportionality between the degree of birefringence $\Delta n$ and the percentage of hexagonality $\alpha$ (Brafman \& Steinberger, 1966). The number of elements, $2 k$, of the Zhdanov symbol ( $a_{1} b_{1} a_{2} b_{2} \ldots a_{k} b_{k}$ ) can be calculated from $\alpha$ and $m$ by the relation

$$
\begin{equation*}
2 k=\alpha m / 100 \tag{5}
\end{equation*}
$$

Thus the birefringence measurements restricted the possible sequences to those with a given number of elements, $2 k$; only these layer sequences were enumerated by the computer.
(b) The calculation of the intensities was performed under the assumption that the structure of the $\mathrm{Zn}-\mathrm{S}$ double layers is the same as in the hexagonal $2 L$ (wurtzite) structure ( $a=3.82 \AA$ ), and that the distance between adjacent Zn layers is also the same as in wurtzite $(3 \cdot 13 \AA)$. In the following, $c$ is calculated from

$$
c=m \times 3.13 \AA
$$

For the calculation of the intensities $I_{h k l}$, only the structure factors and the Lorentz correction were taken into account:

$$
\begin{align*}
& I_{h k l} \propto\left(f_{\mathrm{Zn}}^{2}+f_{\mathrm{S}}^{2}+2 f_{\mathrm{Zn}} f_{\mathrm{S}} \cos 2 \pi l p\right)\left(A_{\mathrm{Zn}}^{2}+B_{\mathrm{Zn}}^{2}\right) \\
& \times\left(1+\cos ^{2} 2 \theta\right) / \sin 2 \theta . \tag{6}
\end{align*}
$$

Here $f_{\mathrm{Zn}}$ and $f_{\mathrm{S}}$ are the zinc and sulphur atomic structure factors. $p=3 /(4 m)^{*}, \theta$ is the Bragg angle. $A_{\mathrm{Zn}}$ and $B_{\mathrm{Zn}}$ are given by

$$
\begin{align*}
& A_{\mathrm{Zn}}=\Sigma \cos 2 \pi(h x+k y+l z)  \tag{7a}\\
& B_{\mathrm{Zn}}=\Sigma \sin 2 \pi(h x+k y+l z) \tag{7b}
\end{align*}
$$

where the summations are over the Zn atoms in the unit cell and $x y z$ denote the coordinates of these atoms.

* In all ZnS polytypes the coordinates of the S atoms are those of the Zn atoms $+(00 p)$.


## Results

## 10L polytype

The ( $10 . l$ ) column of the X-ray oscillation photograph around the $c$ axis is reproduced in Fig. 1. Note in this and in the following photographs the absence of 'smearing out' between the spots; it follows that no one-dimensional stacking disorder was evident in the uniform regions investigated. Sixteen $10 L$ polytypes, differing in layer sequence, may exist; all corresponding intensities were calculated. Table 1 shows the comparison of (10.l) observed intensities with that of the sequence (82). It is seen that the fit is very good. None of the other sequences gave even an approximate fit. The Zhdanov symbol of the observed polytype is thus (8 2) and the $A B C$ sequence (Ramsdell, 1947) is

## ABCABCABAC

On hexagonal axes

$$
a=3 \cdot 82, \quad c=31 \cdot 3 \AA
$$

The hexagonal unit cell has

$$
\begin{array}{llllll}
10 \mathrm{Zn} \text { at } & 0,0,0 & r, s, z & s, r, 2 z & 0,0,3 z & r, s, 4 z \\
& s, r, 5 z & 0,0,6 z & r, s, 7 z & 0,0,8 z & s, r, 9 z
\end{array}
$$

where $z=\frac{1}{10}, r=\frac{2}{3}, s=\frac{1}{3}$.
Percentage of hexagonality: $\alpha=20$. Birefringence at $5460 \AA: \Delta n=4.8 \times 10^{-3}$ (Brafman \& Steinberger, 1966).

## $14 L$ polytype

The (10.l) column is reproduced in Fig. 2. Table 2 shows the comparison of ( $10 . l$ ) observed intensities with that of the only fitting sequence (5423). Thus the $A B C$ sequence is

## $A B C A B A C B A B C B A C$.

On hexagonal axes

$$
a=3.82, \quad c=43.82 \AA
$$

The hexagonal unit cell has

$$
\begin{array}{lllll}
\text { 14Zn at } & 0,0,0 & r, s, z & s, r, 2 z & 0,0,3 z \\
& r, s, 4 z & 0,0,5 z & s, r, 6 z & r, s, 7 z
\end{array}
$$

Table 1. Comparison of calculated and observed (10.l) intensities for the $10 L$ (8 2) polytype

| $l$ | Calc. | Obs. | $l$ | Calc. | Obs. | Further observed relations between intensities |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 5912 | $s$ |  |  |  |  |
| , | 9556 | vs | I | 1598 | vw | $3>4$ |
| 2 | 11677 | ns | $\frac{2}{3}$ | 914 | vw | $7,6>1,2,3$ |
| 3 | 12018 | vs | 3 | 153540 | vos | $7,6<4,5,5$ |
| 4 | 11225 | vs | 4 | 67115 | vos | $\overline{1}>\overline{2}$ |
| 5 | 10828 | vs | 5 | 10828 | vs |  |
| 6 | 20748 | vs | 6 | 3470 | $s$ |  |
| 7 | 25882 | vs | 7 | 2012 | vw |  |
| 8 | 161 | $a$ | 8 | 2055 | $w$ |  |
| 9 | 355 | vw | 9 | 2121 | $w$ |  |

$$
\begin{array}{llll}
0,0,8 z & r, s, 9 z & s, r, 10 z & r, s, 11 z \\
0,0,12 z & s, r, 13 z
\end{array}
$$

where $z=\frac{1}{14}, r=\frac{2}{3}, s=\frac{1}{3}$.
Percentage of hexagonality: $\alpha=28 \cdot 6$.
Birefringence: $\Delta n=7 \cdot 1 \times 10^{-3}$ (Brafman \& Steinberger, 1966).

Table 2. Comparison of calculated and observed (10.l) intensities for the $14 L\left(\begin{array}{l}5 \\ 4\end{array} 2\right.$ 3) polytype

| $l$ | Calc. | Obs. | $l$ | Calc. | Obs. |
| ---: | ---: | :--- | :--- | ---: | :--- |
| 0 | 4112 | $w$ |  |  |  |
| 1 | 1630 | $v w$ | $\overline{1}$ | 13267 | $v s$ |
| 2 | 9903 | $v s$ | $\frac{2}{2}$ | 8056 | $s$ |
| 3 | 19379 | $v s$ | 3 | 29630 | $v s$ |
| 4 | 100916 | $v s$ | $\frac{4}{4}$ | 42963 | $v s$ |
| 5 | 3592 | $v w$ | $\frac{5}{2}$ | 91644 | $v s$ |
| 6 | 70202 | $v s$ | 6 | 14367 | $v s$ |
| 7 | 22332 | $v s$ | 7 | 22978 | $v s$ |
| 8 | 6845 | $w$ | $\frac{8}{8}$ | 33444 | $v s$ |
| 9 | 25081 | $v s$ | $\frac{9}{10}$ | 983 | $a$ |
| 10 | 9746 | $s$ | $\overline{10}$ | 22877 | $v s$ |
| 11 | 7642 | $w$ | $\overline{11}$ | 4968 | $v w$ |
| 12 | 2685 | $a$ | $\frac{1}{12}$ | 3300 | $v w$ |

## $24 L$ polytype

The (10.l) column is represented in Fig.3. The distances between the spots are those corresponding to an 8-layer polytype but they are situated asymmetrically with respect to the zero layer; the zero layer divides the distance between spots neighbouring to it in the ratio $1: 2$. The structure is thus of rhombohedral symmetry. For the layer sequence determination it was thus sufficient to take into account the rhombohedral $24 L$ polytypes only.

Table 3 shows the comparison of (10.l) observed intensities with the only fitting sequence $\left(\begin{array}{ll}5 & 3\end{array}\right)_{3}{ }^{*}$. The $A B C$ sequence is

## ABCABACBCABCACBABCABCBAC

On hexagonal axes $a=3 \cdot 82, c=75 \cdot 12 \AA$.
The hexagonal unit cell has
8 Zn at $\quad 0,0,0 \quad 0,0,3 z \quad 0,0,5 z \quad 0,0,9 z$

$$
0,0,12 z \quad 0,0,15 z \quad 0,0,18 z \quad 0,0,22 z
$$

8 Zn at $\frac{1}{3}, \frac{2}{3}, \frac{1}{3}$ plus the above coordinates
8 Zn at $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$ plus the above coordinates
where $z=\frac{1}{24}$.
Percentage of hexagonality: $\alpha=25$.
Birefringence: $6 \cdot 1 \times 10^{-3}$ (Brafman \& Steinberger, 1966).

## $26 L$ polytype

The ( $10 . l$ ) column is represented in Fig.4. The number of possible $26 L$-layer sequences being too

[^0]Table 3. Comparison of calculated and observed (10.l) intensities for the $24 L(53)_{3}$ polytype

| $l$ | Calc. | Obs. | $l$ | Calc. | Obs. |
| ---: | ---: | :--- | ---: | ---: | :--- |
| 2 | 27641 | $w$ | $\frac{1}{4}$ | 7207 | $v w$ |
| 5 | 127953 | $v s$ | 93246 | $s$ |  |
| 8 | 507616 | $v v s$ | $\underline{7}$ | 175818 | $v s$ |
| 11 | 139621 | $v s$ | $\underline{10}$ | 162029 | $v s$ |
| 14 | 69675 | $s$ | $\overline{13}$ | 90019 | $s$ |
| 17 | 40452 | $w$ | $\overline{16}$ | 126572 | $v s$ |
| 20 | 28581 | $w$ | $\overline{19}$ | 33938 | $w$ |
| 23 | 3543 | $a$ | $\underline{22}$ | 11633 | $w$ |

large, the number of those to be calculated was restricted by considering only Zhdanov sequences compatible with $\alpha$-values obtained from the observed birefringence. Measurement yielded that $\Delta n=3.6 \times$ $10^{-3}$. This is near to $\Delta n=3.7 \times 10^{-3}$ calculated from $\alpha=15.4(k=2)$. Clearly other values of $k$ can be disregarded, as $k=1$ would yield $\Delta n=1.85 \times 10^{-3}$ and $k=3 \Delta n=5.54 \times 10^{-3}$. Thus for the comparison only Zhdanov sequences with 4 elements had to be considered. Table 4 shows the comparison of (10.l) observed intensities with those of the only fitting sequence (17423). The $A B C$ sequence is

## ABCABCABCABCABCABACBABCBAC.

On hexagonal axes

$$
a=3.82, \quad c=81.38 \AA .
$$

The hexagonal unit cell has

$$
\begin{array}{rlllll}
26 \mathrm{Zn} \text { at } & 0,0,0 & r, s, z & s, r, 2 z & 0,0,3 z & r, s, 4 z \\
& s, r, 5 z & 0,0,6 z & r, s, 7 z & s, r, 8 z & 0,0,9 z \\
r, s, 10 z & s, r, 11 z & 0,0,12 z & r, s, 13 z & \\
& s, r, 14 z & 0,0,15 z & r, s, 16 z & 0,0,17 z & \\
& s, r, 18 z & r, s, 19 z & 0,0,20 z & r, s, 21 z & \\
& s, r, 22 z & r, s, 23 z & 0,0,24 z & s, r, 25 z &
\end{array}
$$

where $z=\frac{1}{26}, r=\frac{2}{3}, s=\frac{1}{3}$.

## $28 L$ polytype

The ( $10 . l$ ) column is represented in Fig. 5. The number of 28 -layer sequences to be compared was restricted as in the case of $26 L$. For $k=1 \Delta n=1.71 \times$ $10^{-3}$; for $k=2 \Delta n=3.43 \times 10^{-3}$; for $k=3 \Delta n=5 \cdot 14 \times$ $10^{-3}$. The measured value of $\Delta n$ was $3 \cdot 6$. Thus $k=2$; only Zhdanov sequences with four elements had to be considered. Table 5 shows the comparison of ( $10 . l$ ) observed intensities with those of the only fitting sequence (9559). The $A B C$ sequence is

## ABCABCABCBACBABCABCBACBACBAC.

On hexagonal axes

$$
a=3.82, \quad c=87.64 \AA .
$$

The hexagonal unit cell has


Fig. 1. (10.l) column of an oscillation photograph around the $c$ axis of the 10 L polytype. $\mathrm{Cu} K \alpha$ radiation, 60 mm diameter camera $(\times 3)$. The position of the zero layer is indicated by an arrow.


Fig. 2. As Fig. 1, but for $14 L$ polytype.


Fig. 3. As Fig. 1, but for $24 L$ (rhombohedral) polytype.


Fig.4. As Fig.1, but for $26 L$ polytype.

Fig. 5. As Fig. 1, but for $28 L$ polytype.

$$
\begin{aligned}
& 28 \mathrm{Zn} \text { at } 0,0,0 \quad r, s, z \quad s, r, 2 z \quad 0,0,3 z \quad r, s, 4 z \\
& s, r, 5 z \quad 0,0,6 z \quad r, s, 7 z \quad s, r, 8 z \quad r, s, 9 z \\
& 0,0,10 z \quad s, r, 11 z \quad r, s, 12 z \quad 0,0,13 z \\
& r, s, 14 z \quad s, r, 15 z \quad 0,0,16 z \quad r, s, 17 z \\
& s, r, 18 z \quad r, s, 19 z \quad 0,0,20 z \quad s, r, 21 z \\
& r, s, 22 z \quad 0,0,23 z \quad s, r, 24 z \quad r, s, 25 z \\
& 0,0,26 z \quad s, r, 27 z
\end{aligned}
$$

where $z=\frac{1}{28}, r=\frac{2}{3}, s=\frac{1}{3}$.
Space does not permit publication of the intensity distributions of the several hundred layer sequences of the various polytypes calculated by the computer. It should, however, be emphasized that except for the layer sequences given above there were no other layer sequences in any case which gave even a rough fit with the observed intensities.

## Discussion

The present work demonstrates that the use of birefringence results makes it possible to shorten the calculations leading to identification of ZnS polytype layer structures. The linear relation used between the degree of birefringence and percentage of hexagonality had been obtained from data on simple structures (Brafman \& Steinberger, 1966); the validity of this relation is further substantiated by the excellent fit of the intensities in the case of $26 L$ and $28 L$ polytypes. The calculations leading to the identification of layer sequences are, however, too lengthy in the case of the higher polytypes even if the birefringence results are taken into account. In fact, the structures of 40 L and $120 L$ polytypes previously mentioned (Brafman \&

Table 4. Comparison of calculated and observed (10.l) intensities for the $26 L(1742$ 3) polytype

| $l$ | Calc. | Obs. | $l$ | Calc. | Obs. | Further observed relations between intensities |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 3932 | vw |  |  |  |  |
| 1 | 2718 | $a$ | İ | 5168 | vw |  |
| 2 | 2101 | $a$ | $\overline{2}$ | 13843 | w | $9>8$ |
| 3 | 9926 | $w$ | 3 | 13138 | $w$ | $7>8$ |
| 4 | 6568 | vw | 4 | 7661 | vw | $11 \simeq 12<8$ |
| 5 | 1670 | $a$ | 5 | 21841 | $s$ | $5 \sim 6<6$ |
| 6 | 42268 | $s$ | 6 | 24890 | $s$ | $14>1 \overline{3} \simeq 5$ |
| 7 | 94833 | vs | 7 | 9274 | vw | $\overline{2} \simeq 3>3$ |
| 8 | 73463 | vs | 8 | 295861 | vos |  |
| 9 | 11717 | $w$ | 9 | 640129 | vos |  |
| 10 | 14231 | $w$ | 10 | 8678 | vw |  |
| 11 | 62950 | vs | $\overline{11}$ | 16036 | $w$ |  |
| 12 | 64292 | vs | $\overline{12}$ | 3304 | $a$ |  |
| 13 | 22151 | $w$ | $\overline{13}$ | 22150 | $s$ |  |
| 14 | 2203 | $a$ | $\overline{14}$ | 42862 | $s$ |  |
| 15 | 7318 | vw | $\overline{15}$ | 28727 | $s$ |  |
| 16 | 2872 | $a$ | $\overline{16}$ | 4710 | $a$ |  |
| 17 | 168364 | vs | $\overline{17}$ | 3081 | $a$ |  |
| 18 | 69243 | $s$ | $\overline{18}$ | 17193 | vw |  |
| 19 | 2151 | $a$ | $\overline{19}$ | 21995 | $w$ |  |

Table 5. Comparison of calculated and observed (10.l) intensities for the $28 L$ (9 559 ) polytype
Further observed

* The observed intensities were symmetrical with respect to the zero line $(l=0)$.

Steinberger, 1966) have for this reason not yet been determined.

Comparison of published ZnS polytype layer sequences with those known in SiC immediately demonstrates the fact that the numbers of elements in the Zhdanov symbols of ZnS polytypes are, in general, smaller than those reported in SiC polytypes. In other words, the percentage of hexagonality of ZnS polytypes tends to be lower.

For SiC polytypes, the theory which explains the growth of most observed polytypes is that of Frank (1961) and Mitchell (1957), as modified by Krishna \& Verma (1965). This theory is based on the fact that in SiC a thin platelet with ( 00.1 ) faces is at first formed. The platelet may have one of the structures $4 L, 6 L$ or $15 L$ (these being the most frequent SiC structures). As a result of buckling, a step appears. Its height is usually less than the $c$ dimension of the unit cell of the basic structure. A screw dislocation is thus formed which ensures further growth; the Burgers vector of this dislocation is a non-integral multiple of the basic $c$ translation.

It is very unlikely that for polytypism in ZnS the above model is correct in its present form. First, in the initial stages of the growth thin needles form with the axis [ $00 \cdot 1$ ]; such needles cannot buckle in the way proposed for platelets. Secondly, among the observed

ZnS polytypes there are some which have large elements in their Zhdanov sequence [e.g. (17 42 2 3 )], whereas on the basis of the Mitchell-Verma model such large elements can not be expected.

A modification of the above model, or else a quite different one is, therefore, needed for the explanation of polytypism in ZnS . It seems to be, however, premature to contemplate such a model, since the number of known ZnS polytypes is still too low for meaningful comparison of experiment with theory.

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# The Crystal Structure of Adamantane: An Example of a False Minimum in Least Squares 

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A least-squares refinement of the structure of the low temperature form of adamantane, starting with molecules having symmetry 43 m , and tilted $9^{\circ}$ about the $c$ axis, refines to a structure not significantly different, and a reduction of the residual from 3.92 to $3 \cdot 11$. On the other hand, if the refinement is started at the published structure, the parameters of which correspond to rather distorted molecules, a structure not significantly different from this starting point results, and a reduction of the residual from $15 \cdot 49$ to $14 \cdot 34$.

## Introduction

A recent study of the crystal structure of the low temperature phase of adamantane (Nordman \& Schmitkons, 1965, hereinafter referred to as NS) in which it was reported that the molecule departed significantly from $\overline{4} 3 m$ symmetry by being compressed along one of its $\overline{4}$ axes, prompted us to examine this structure

[^1]further, partly because NS stated that 'there is no readily apparent interpretation of this effect in terms of close intermolecular contacts', and partly because of current interest in these laboratories in this phase and its relation to the high temperature phase (Dows \& Rubenstein, 1965). The low temperature phase, according to NS, is tetragonal, space group $P \overline{4} 2_{1} c, a=$ $6 \cdot 60, c=8 \cdot 81 \AA, Z=2$. Precession data ( $\mathrm{Cu} K \alpha, 53$ visually estimated $F_{\text {obs }}$ ) were refined by least squares, optimizing seven positional parameters for the carbon atoms, twelve positional parameters for the hydrogen


[^0]:    * This is the only polytype structure reported in the present work which has an SiC counterpart (Gomes de Mesquita, 1965).

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